Fractional Statistics and the Butterfly Effect

(Based on JHEP 2016(8) 129 [arXiv:1602.06543] with Xiao-Liang Qi)

Yingfei Gu

Harvard University

UIUC, Nov 4, 2017

Outline

- Introduction: late time regime of out-of-time-ordered correlation function (OTOC);
- Chaos is related to topology via OTOC;
- Some comments on the results: level rank dualiy; averaged OTOC and topological entanglement entropy;
- Summary.

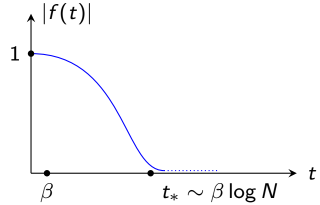
Introduction

- Thermalization in quantum many-body systems;
- Quantum butterfly effect diagnosed by OTOC (Kitaev, Shenker, Stanford...):

$$\langle W^\dagger(t) V^\dagger(0) W(t) V(0) \rangle_\beta$$

  Normalized OTOC $f(t) := \frac{\langle \widetilde{W}^\dagger V^\dagger \widetilde{W} V \rangle_\beta}{\sqrt{\langle \widetilde{W}^\dagger V^\dagger V \widetilde{W} \rangle_\beta \langle V^\dagger \widetilde{W}^\dagger \widetilde{W} V \rangle_\beta}}$.
  $\widetilde{W} := W(t)$, bounded norm $|f(t)| \leqslant 1$

- For example SYK model (sketch):



Early time v.s. late time regime

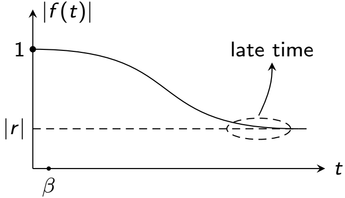
- Many studies focus on early time regime: how chaos develops, Lyapunov behavior ... ;

- Late time regime, sufficient scrambling:

$$\lim_{t\to\infty} f(t) = 0$$

  assuming $\langle W \rangle = \langle V \rangle = 0$ and large total Hilbert space;

- This talk: see how "effective conservation law" prevents sufficient scrambling.



OTOC in RCFTs

- System: $(1+1)$-d rational conformal field theories
  - Examples: Ising CFT, WZW models ....
  - Algebraic data: modular tensor category (Moore, Seiberg, ... 1980s) F, R matrices, modular S-matrix;
- Pick a pair of primary fields W and V, express OTOC in terms of conformal blocks (Roberts, Stanford 2015);
- Take long time limit $t \gg \beta$, we found universal late time behavior determined by scaling dimensions and F, R matrices.

OTOC in RCFTs continued

- In particular, residue value at $t \to \infty$ is determined by **modular S-matrix**: $S = (s_{wv})$

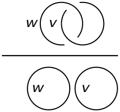
$$r[w,v] := \lim_{t\to\infty} f(t) = \frac{s_{11}s^*_{wv}}{s_{1w}s_{1v}} \left(= \frac{\mathcal{D}s^*_{wv}}{d_w d_v} \right)$$

w, v are family labels for primary fields W, V. $d_{w/v}$: quantum dimensions, $\mathcal{D}$: total quantum dimension.

Relation to TQFTs

The result has a pictorial expression by identifying modular S-matrix with topological S-matrix in topological field theory:

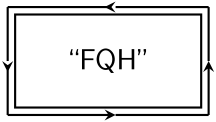
$$r[w,v] = \frac{\mathcal{D} s^*_{wv}}{d_w d_v} =$$

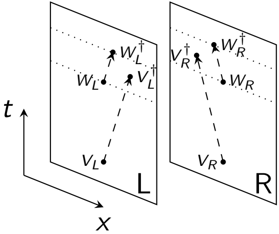


We can show this using the following ingredients:

- Bulk-boundary correspondence : $(2+1)$-d TQFT/ $(1+1)$-d RCFT (Witten 1989 ...);
- Factorization: $\langle \widetilde{W}^\dagger V^\dagger \widetilde{W} V \rangle = \sum_i (\text{Left})_i \cdot (\text{Right})_i$
- Idea: move the chiral part from out-of-time ordred to time ordered.

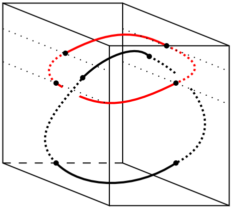
e.g.

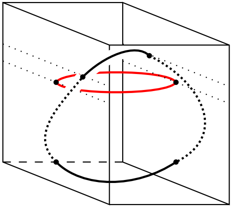




Bulk picture

Anyon world-lines in bulk (Red/black lines: anyon w/v)





- OTOC: $\langle \widetilde{W}^\dagger V^\dagger \widetilde{W} V \rangle_\beta$
- *Linking* of anyon world-lines
- TQFT value $\mathcal{D} s^*_{wv}$

- KS ordered: $\langle V^\dagger \widetilde{W}^\dagger \widetilde{W} V \rangle_\beta$
- *No linking*
- TQFT value $d_w d_v$.

Topological nature of OTOC: bulk linking of anyon world-lines.

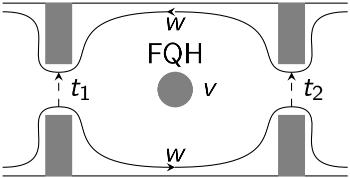
Some comments on $r[w, v]$

- Interference in fusion channel $w \times v = \sum_u u$ suppresses $|r[w,v]| = \frac{\mathcal{D}|s_{wv}|}{d_w d_v} \leqslant 1$:

$$\mathcal{D}|s_{wv}| = \left| \sum_u d_u N_{wv}^u \underbrace{\frac{\theta_u}{\theta_w \theta_v}}_{\text{pure phase}} \right| \leqslant \sum d_u N_{wv}^u = d_w d_v$$

- $r[w,v] = \frac{\mathcal{D}s_{wv}^*}{d_w d_v}$ coincides with "monodromy scalar" in the anyon interferometry experiments:

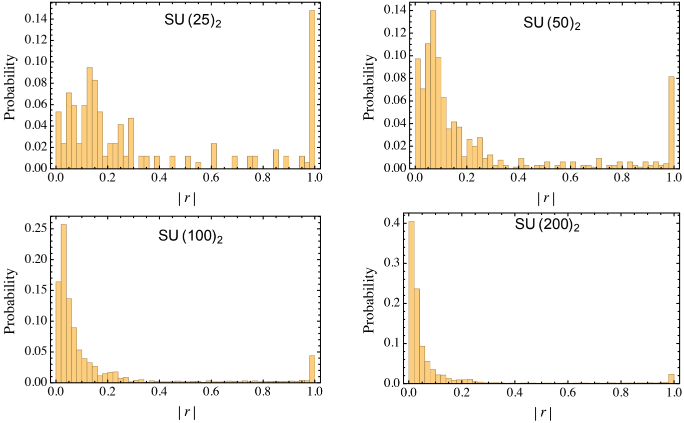
e.g.



Further comments: $|r[w, v]|$ histogram

$r[w, v]$ measures how "effective conservation law" (topological order) prevents sufficient scrambling.

- Example: histogram of $|r[w, v]|$ in $SU(N)_2$ theories when sampling over all primaries:



A corollary from level rank duality

- As expected, roughly speaking a more "complicated" theory (more primaries, larger central charge $\sim 2N$...) scrambles more sufficiently;
- We prove a statement based on level-rank duality:

Histogram of $|r[w, v]|$ for $SU(N)_2$ is identical to $SU(2)_N$

- $SU(2)_N$ in large N is weak coupling. Sufficient scrambling doesn't require strong coupling?

Averaged late time OTOC

- Randomly pick an operator, the chance of family w:

$$p_w \propto \dim(\mathcal{H}_w \otimes \mathcal{H}_{\overline{w}}) \propto d_w^2, \quad \Rightarrow \quad p_w = \frac{d_w^2}{\mathcal{D}^2}$$

- Include the phase of $r[w, v]$;
- The averaged $r[w, v]$:

$$\boxed{\langle r \rangle := \sum_{w,v} p_w p_v r[w, v] = \frac{1}{\mathcal{D}^2}}$$

Remark: Topological entanglement entropy (Kitaev, Preskill; Levin, Wen 2005)

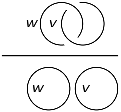
$$\gamma = \log \mathcal{D} \Rightarrow \langle r \rangle = e^{-2\gamma}$$

Summary

- Late time regime of OTOC: "effective conservation law" prevents sufficient scrambling;

- Topological meaning of late time OTOC:

$$r[w, v] = \frac{\mathcal{D} s^*_{wv}}{d_w d_v} =$$



;

- Some comments:
  - Suppression of $|r|$ from interference;
  - Histogram: $\mathrm{SU}(2)_N$ and $\mathrm{SU}(N)_2$ (level-rank duality);
  - Random average of residue value: $\langle r \rangle = 1/\mathcal{D}^2 = e^{-2\gamma}$.

$\Rightarrow$ Intrinsic relation between the butterfly effect in $(1+1)$-d RCFTs and fractional statistics in $(2+1)$-d.